Vaulting and Displacement of Pole-Covered 180°-Domain Walls

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(Z. Naturforsch. **29 a**, 457–461 [1974]; received 1 April 1973)

A zero approximation has been given of the normalized vaulting amplitudes of 180°-domain walls which are assumed to be vaulted between repulsive interaction lines if equilibrium has taken place between an external magnetic field and the counteracting forces resulting from the stray fields of the vaulting-induced free magnetic surface poles. Calculations have been carried out for two cases: cylindrical vaulting between linear barrier lines and pillow-shaped vaulting between isolated interaction points building up a tridimensional lattice of hindering points.

§ 1. Introduction

First investigations concerning the possibility of domain wall vaulting had been carried out by Neel [1] in 1946. He was followed by Berner [2], and by Kersten [3, 4] who was the first to base a theory of the initial susceptibility on the mechanism of domain wall vaulting. Dietze [5] succeeded in extending Kersten's model, taking into consideration also the case of 90°-domain walls covered by vaulting-induced free magnetic poles and thus causing counteracting stray fields. These studies however, having mainly dealed with 90°-domain walls, gradually fell into oblivion when the more successful potential theory was developed.

Meanwhile however by Blank [6], Cullity and Allen [7], Markert [8–11], Markert and Steigenberger [12], Markert, Scherber and Wagner [13], Markert, Steigenberger, and Wagner [14], Hayashi [15, 16], Hayashi, Takahashi, and Yamamoto [17, 18], Chebotkevich, Urusovskaya, and Veter [19], and Hayashi and Yamamoto [20], a series of experimental observations and theoretical investigations had been published which only very hardly could be understood in terms of the usual potential theory. Therefore attempts have been started to develop an alternative – or at least a rather modified – model [8–10, 12–14, 16] one of the main ideas of which should be to apply the earlier conceptions of domain wall vaulting on pole covered 180°-domain walls [9, 10].

The mean amplitudes of 180°-domain wall vaulting against vaulting-induced counteracting stray fields however are usually expected to be negligibly small. Contrary to that it is the aim of the present study to show that the normalized vaulting amplitude depends linearly on the ratio $J_S/|K_1|$ of saturation magnetization over crystalline anisotropy constant, i. e. on a quantity which, within suitable temperature ranges, increases in many magnetic materials up to rather high magnitude.

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§ 2. Magnitude of Cylindrical Vaulting

For application in the theory of magnetization processes it is sufficient to have at least an estimation of the order of magnitude up to which the vaulting amplitudes really will increase. So we can simplify the problem to the linear one illustrated in Figure 1: between two lines A and B (sectional

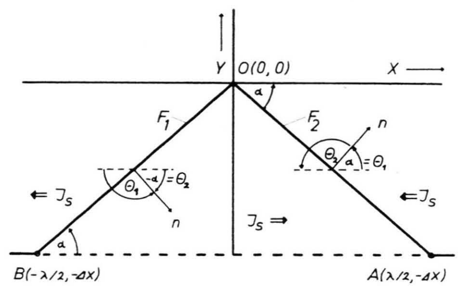

Fig. 1. Sectional view of a 180°-domain wall segment being prismatically vaulted between two parallel bordering lines A and B (normal to the drawing plane). The vaulting amplitude is Δx, the side-faces have areas F_1 and F_2, respectively.

view) of distance λ and coordinates $(\lambda/2;\ -\Delta x)$ and $(-\lambda/2;\ -\Delta x)$, respectively, a 180°-domain wall is vaulted prismatically to an amplitude Δx. In this case the equilibrium position obviously is defined by energy minimization

$$\partial\,[E_J + E_W + E_H]/\partial\,(\Delta x) = 0\,, \qquad (1)$$

where E_J means the stray field energy due to the free magnetic poles occuring on the wall areas F_1 and F_2, E_W denotes the wall energy and E_H designates the potential energy of an external magnetic field H_{ex} aligned parallel to the x-axis. As E_H and E_W simply are given by

$$E_H = -2 J_s H_{ex} b(\lambda/2) \cdot \Delta x \tag{2}$$

and

$$E_W = 2\gamma_W b l = 2\gamma_W b(\lambda^2/4 + (\Delta x)^2)^{1/2}, \tag{3}$$

respectively, with b designating the length of the lines A and B in z-direction, i. e. normal to the drawing plane of Figure 1, with γ_W meaning the specific domain wall energy, and with J_s designating the saturation magnetization, our problem reduces to an estimation of the stray field energy

$$E_J = -(1/2) \int_{\text{prism}} \boldsymbol{J}_s \boldsymbol{H}_{st} \, dv \tag{4}$$

where the stray field H_{st} is given by

$$H_{st} = -\operatorname{grad} \Psi, \tag{5}$$

and Ψ means the solution of Poisson's equation

$$\Delta \Psi = 4\pi \operatorname{div} J_s = -4\pi\sigma_m, \tag{6}$$

i. e.

$$\Psi = -\int((\operatorname{div} J_s)/r)\, dF = \int(\sigma_m/r)\, dF, \tag{7}$$

with σ_m designating the magnetic pole density on the wall areas F_1 and F_2, and with

$$r = ((x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2)^{1/2} \tag{8}$$

denoting the distance between

$$dF = (\cos\alpha)^{-1}\, dz\, dx \tag{9}$$

and a space point (x_0, y_0, z_0) inside of the prism A0B. To formulate Eq. (7) more explicitly, let us start from

$$\Psi = \int_0^{F_1} (\sigma_{m_1}/r)\, dF + \int_{F_1}^{(F_1+F_2)} (\sigma_{m_2}/r)\, dF. \tag{10}$$

As $\sigma_{m_1} = \text{const.}$, $\sigma_{m_2} = \text{const.}$, substitution of Eq. (9) leads to

$$\Psi = (\sigma_{m_1}/\cos\alpha) \int_{-b/2}^{+b/2}\int_{x=-\lambda/2}^{x=0} r^{-1}\, dz\, dx + (\sigma_{m_2}/\cos\alpha)\int_{z=-b/2}^{z=+b/2}\int_{x=0}^{x=\lambda/2} r^{-1}\, dz\, dx. \tag{11}$$

The next step is to simplify Equation (8). As in Eqs. (10) and (11) integration has to be carried out over the wall areas F_1 and F_2, respectively, it follows that for the coordinates x and y occuring in Equation (8), there holds a relationship

$$\begin{cases} y = x \operatorname{tg}\alpha \approx x\alpha, & \text{if } -\lambda/2 \leq x \leq 0, \\ y = -x\operatorname{tg}\alpha \approx -x\alpha, & \text{if } 0 \leq x \leq \lambda/2. \end{cases} \tag{12}$$

If we further postulate b to be sufficiently long, we can assume that in every cross section of the prism A0B the $\Psi(x_0, y_0, z_0)$ are the same ones as in the particular cross section characterized by $z_0 = 0$, i. e. we can assume $\Psi(x_0, y_0, z_0)$ to be nearly independent of z_0. Thus, if we additionally take into account that

$$\sigma_{m_1} = -2J_s\alpha \quad \sigma_{m_2} = +2J_s\alpha, \tag{13}$$

Eq. (11) varies to

$$\Psi = -2J_s\alpha \int_{z=-b/2}^{z=+b/2}\int_{x=-\lambda/2}^{x=0} ((x-x_0)^2 + (\alpha x - y_0)^2 + z^2)^{-1/2}\, dz\, dx + 2J_s\alpha\int_{z=-b/2}^{z=+b/2}\int_{x=0}^{x=\lambda/2}((x-x_0)^2 + (\alpha x + y_0)^2 + z^2)^{-1/2}\, dz\, dx. \tag{14}$$

Although the denominators of the above integrals become zero if taken along the straight lines $y_0 = \alpha x_0$ and $y_0 = -\alpha x_0$, respectively, the corresponding integrals must converge because it is possible to find convergent major functions of them. But despite that convergence, inside the prism A0B the function $\Psi(x_0, y_0)$ cannot be represented by any analytic function but can only be estimated by analytic major functions of its two integrals.

Fortunately it can be shown that for calculation of E_J it is sufficient to know $\Psi(x_0, y_0)$ along the straight lines $y_0 = \alpha x_0$ and $y_0 = -\alpha x_0$, respectively. The reason for this is as follows: if we assume the crystalline anisotropy, having an easy axis parallel to the x-axis, to be sufficiently high, the saturation magnetization inside the prism has the components

$$\boldsymbol{J}_s = (J_s; 0; 0). \tag{15}$$

Thus, according to Eq. (4), we get

$$E_J(\alpha) = -(J_s/2)\int_{\text{prism}} (H_{st})_x\, dV, \tag{16}$$

and due to Eq. (5) this becomes

$$E_J(\alpha) = (J_s/2)\int_{z_0=+b/2}^{z_0=+b/2}\int_{x_0=+y_0/\alpha}^{x_0=-y_0/\alpha}\int_{y_0=-\Delta x}^{y_0=0}(\partial\Psi(x_0,y_0)/\partial x_0) \cdot dz_0\, dx_0\, dy_0. \tag{17}$$

As integration with respect to z_0 simply can be carried out, we get

$$E_J(\alpha) = (b J_s/2)\int_{x_0=+y_0/\alpha}^{x_0=-y_0/\alpha}\int_{y_0=-\Delta x}^{y_0=0}(\partial\Psi(x_0,y_0)/\partial x_0)\, dx_0\, dy_0. \tag{18}$$

After integration with respect to x_0, our problem reduces to

$$E_J(a) = (b\,J_s/2) \int\limits_{y_0=-\Delta x}^{y_0=0} [\Psi(-y_0/a, y_0) - \\ - \Psi(+y_0/a, y_0)]\,\mathrm{d}y_0 . \tag{19}$$

The final integration with respect to y_0, although somewhat complicated, is not really difficult. It has been carried out by H. Markert [9] and shall not be repeated here in detail. The result is that $E_J(\alpha)$ can be represented by

$$E_J(a) = b\,J_s \int\limits_{y_0=-\Delta x}^{y_0=0} \Psi(-y_0/a, y_0)\,\mathrm{d}y_0 , \tag{20}$$

where $\Psi(-y_0/\alpha, y_0)$ is estimated to

$$\Psi(-y_0/a, y_0) \approx 2\,J_s\,a \;\cdot \\ \cdot\; \{(-4\,y_0/a)\,[1 - \ln{(2\,y_0/a\,\lambda)}]\} . \tag{21}$$

Equations (20) and (21) then lead to

$$E_J(a) = (3/2)\,J_s^2\,a^2\,b\,\lambda^2 \tag{22}$$

and because of $a = 2\,\Delta x/\lambda$ the stray field energy amounts to

$$E_J(\Delta x) = 6\,J_s^2\,\Delta x^2\,b . \tag{23}$$

Till now we calculated for the idealized case of infinite crystalline anisotropy energy K_1. If K_1 has a given finite value, Eq. (15) obviously cannot hold true at least within a surface layer of, say, sometimes the lattice parameter in thickness. To correct that effect, according to Shockley [21], Williams, Bozorth and Shockley [22], and due to Kneller [23], we have to take into account the so-called μ^*-correction, getting then

$$E_J^*(\Delta x) = E_J(\Delta x)\cdot 2/(1+\mu^*) , \tag{24}$$

with

$$\mu^* = 1 + 4\,\pi\,J_s^2/2\,|K_1| . \tag{25}$$

As for nickel, iron and magnetite at room temperature

$$4\,\pi\,J_s^2/2\,|K_1| \gg 1 , \tag{26}$$

Eq. (24) can be simplified to

$$E_J^*(\Delta x) = (6/\pi)\,|K_1|\,\Delta x^2\,b . \tag{27}$$

Neglecting the wall energy E_W, i. e. assuming $E_W/E_J^*(\Delta x) \ll 1$, and substituting Eqs. (2) and (27) into Eq. (1), we find our final result, the normalized vaulting amplitude to come to

$$\Delta x/\lambda = (\pi/12)\cdot(J_s\,H_{ex}/|K_1|) . \tag{28}$$

This estimation holds true as long as the magnitude of Δx is at least comparable with the domain wall thickness δ_d. If however $\Delta x < \delta_d$, i. e. if

$$(\pi/12)\cdot(H_{ex}\,J_s/|K_1|)\,\lambda < \delta_d , \tag{29}$$

the finite thickness of the domain wall should be taken into account. To do that approximatively, let us simplify the spin configurations and geometrical conditions as illustrated in Figure 2. Equation (6) then has to be replaced by

$$\varrho_m = -\operatorname{div} J = -(\partial J_x/\partial x) , \tag{30}$$

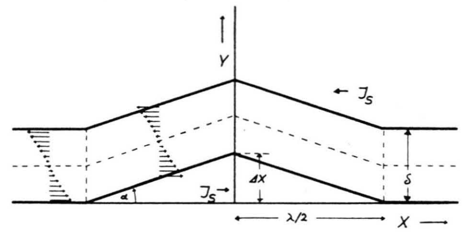

Fig. 2. Sectional view of a prismatically vaulted segment of a 180°-domain wall of finite thickness δ_B, illustrating the occurrence of magnetic space charge in the case of $\delta_B > \Delta x$.

where $J_x(x, y)$ is defined by

$$\begin{aligned} J_x(x,y) &= J_x^{\mathrm{I}}, \quad \text{if} \quad -\lambda/2 \leqq x \leqq 0 \\ J_x(x,y) &= J_x^{\mathrm{II}}, \quad \text{if} \quad 0 \leqq x \leqq \lambda/2 , \end{aligned} \tag{31}$$

and where

$$J_x^{\mathrm{I}} \begin{cases} = J_s, \quad \text{if} \quad 0 \leqq y \leqq a\cdot(x+\lambda/2) \\ = J_s\cdot[(2\,a/\delta_d)\cdot(x+\lambda/2)+1-2\,y/\delta_d], \quad \text{if} \\ \qquad a(x+\lambda/2) \leqq y \leqq a\,(x+\lambda/2)+\delta_d \\ = -J_s, \quad \text{if} \quad [a(x+\lambda/2)+\delta_d] \leqq y , \end{cases} \tag{32 a}$$

$$J_x^{\mathrm{II}} \begin{cases} = J_s, \quad \text{if} \quad 0 \leqq y \leqq a\,(\lambda/2-x) \\ = J_s\cdot[(2\,a/\delta_d)\cdot(\lambda/2-x)+1-2\,y/\delta_d], \quad \text{if} \\ \qquad a(\lambda/2-x) \leqq y \leqq a\,(\lambda/2-x)+\delta_d \\ = -J_s, \quad \text{if} \quad [a(\lambda/2-x)+\delta_d] \leqq y . \end{cases} \tag{32 b}$$

If the further mathematical treatment now is carried out analogously to the way outlined above, according to Markert [9], it yields

$$\Psi(x_0, y_0) = -2\,J_s(a/\delta_d) \int\limits_{-b/2}^{+b/2} \mathrm{d}z \int\limits_{-\lambda/2}^{0} \mathrm{d}x \int\limits_{\alpha(\lambda/2+x)}^{\alpha(\lambda/2+x)+\delta_d} \cdot \\ \cdot\; [(x-x_0)^2 + (y-y_0)^2 + z^2]^{-1/2}\,\mathrm{d}y \;+ \\ + 2\,J_s(a/\delta_d) \int\limits_{-b/2}^{+b/2} \mathrm{d}z \int\limits_{0}^{+\lambda/2} \mathrm{d}y \int\limits_{\alpha(\lambda/2-x)}^{\alpha(\lambda/2-x)+\delta_d} [(x-x_0)^2 + (y-y_0)^2 + \\ + z^2]^{-1/2}\,\mathrm{d}y \tag{33}$$

instead of the former Equation (14). Although Eq. (33) is solvable at least approximately, we shall not insist on solving it really but confine us to the qualitative remark that, according to Fig. 2, in the case of $\Delta x \ll \delta_d$ the total magnetic space charge of given sign obviously is much smaller than the corresponding magnetic surface charges $\sigma_m F_1$ or $\sigma_m F_2$ resulting from our above estimation. Thus it follows that in this very case $\Delta x \ll \delta_d$ the stray field energy to be calculated from Eq. (33) will be considerably smaller than we would get it from Eq. (27), and, particularly, that $\Delta x/\lambda$, i.e. the normalized vaulting amplitude, will become larger than we would estimate it according to Equation (28).

§ 3. Magnitude of Spherical Vaulting

Again, as in § 2, we simplify the problem to the linearized modification illustrated in Figure 3: the

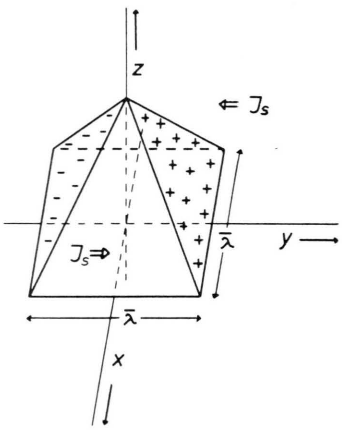

Fig. 3. Sketch of a pyramidally vaulted segment of a 180°-domain wall, illustrating the simplest orientation with respect to the axis of easy magnetization. In this case only two opposite side-faces are covered with free magnetic poles.

estimation of the amplitude of pyramidal vaulting. Thereby we confine the consideration to a pyramidally vaulted segment of a 180°-domain wall, assuming that segment to be oriented in the way shown in Fig. 3, and to have quadratic horizontal projection of edge length $\bar{\lambda}$.

As equilibrium vaulting again is characterized by

$$\partial [E_J^* + E_W + E_H]/\partial (\Delta z/\bar{\lambda}) = 0 , \qquad (34)$$

where $\Delta z/\bar{\lambda}$ designates the normalized vaulting amplitude, the main problem is to estimate the stray field energy E_J^* resulting from the surface density of the free magnetic poles covering two opposite side-faces of the quadratic pyramid. For a somewhat rough calculation. according to Goodenough [24], fortunately we can content ourselves with the following expression

$$E_J(\Delta z, \bar{\lambda}) = 2(2/3)\pi\,\omega^{*2} F_\Delta D , \qquad (35)$$

where the side-face F_Δ is given by

$$F_\Delta = (\bar{\lambda}/2) \cdot (\bar{\lambda}^2/4 + \Delta z^2)^{1/2} , \qquad (36)$$

whilst D designates a mean diameter of F_Δ, i.e.

$$D \approx 2(F_\Delta/\pi)^{1/2} , \qquad (37)$$

and the term

$$\omega^* = J_s(\cos\Theta_1 - \cos\Theta_2) \qquad (38)$$

is a measure of the surface density of the free magnetic poles covering those side-faces F_Δ the normals of which have angles $\Theta_1 \neq 90°$ and $\Theta_2 \neq 90°$ with the direction of easy magnetization $\boldsymbol{J}_s$, i.e. with the y-axis. Thus, as

$$\Theta_2 = 180° - \Theta_1 \qquad (39)$$

and

$$\cos\Theta_1 = \sin(90° - \Theta_1) \approx \mathrm{tg}(90° - \Theta_1) \approx \Delta z/(\bar{\lambda}/2) , \qquad (40)$$

it follows from Eqs. (35) to (40) that E_J can be represented by

$$E_J(\Delta z, \bar{\lambda}) \approx (16\,\pi^{1/2}/3) J_s^2 (\Delta z/\bar{\lambda})^2 [1 + 2(\Delta z/\bar{\lambda})^2] . \qquad (41)$$

Considering additionally the μ^*-correction we get

$$E_J^*(\Delta z, \bar{\lambda}) \approx E_J(\Delta z, \bar{\lambda})\, 2/(1+\mu^*) \approx E_J(\Delta z, \bar{\lambda}) |K_1| / \pi J_s^2 . \qquad (42)$$

If we now neglect the wall energy E_W with regard to the larger terms of E_J^* and of the potential energy

$$E_H = -(2/3) J_s H_{ex} \bar{\lambda}^3 (\Delta z/\bar{\lambda}) , \qquad (43)$$

substitution into Eq. (34) immediately yields

$$\Delta z/\bar{\lambda} \approx (\pi^{1/2}/16) \cdot (J_s H_{ex}/|K_1|) , \qquad (44)$$

i.e. a formula that is very similar to the one represented by Eq. (28) and describing the above result of the corresponding case of cylindrical vaulting.

Acknowledgement

The author is very indebted to Prof. Dr. J. Brandmüller and to Prof. Dr. A. Faessler for their hospitality conceded to him at the 'Fakultät für Physik', University of Munich, where most of the present work has been done, and to Prof. Dr. K. Stierstadt (also at 'Fakultät für Physik', Univers. of Munich) for many stimulating discussions.

Last not least it is gratefully acknowledged that the above investigations have been supported by the Deutsche Forschungsgemeinschaft.

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